A modeling method for the Earth's interior structure based on the principles of \(2^n\) geometry and the resulting Earth's tectonic model

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Abstract. A non-seismic wave detection method based on \(2^n\) geometry principle is proposed to deduce the vertical structure of the earth. By using this method, a new earth structure model can be derived by pure theory, avoiding the traditional earth modeling method that mainly relies on seismic wave imaging. The traditional method is to fill in the new findings and refine the model by studying the detection results of seismic waves, gravity and magnetic field. The deduction method proposed in this paper does not depend on seismic waves, gravity and magnetic field. Using pure geometry theory, it can derive results highly similar to the PREM model (more than 90%), and predict some new structures that can not be actively derived by PREM and other models.

Keywords: Earth structure; Earth modeling; non-PREM model; non-seismic wave modeling.

1. Introduction

Through the continuous development and improvement of seismic wave detection and modeling technology \(^3\), \(^4\), \(^6\), models such as PREM \(^12\), \(^13\) have initially outlined the vertical structure of the earth's interior \(^15\). However, the cause of this structure and the relationship between the various internal layers have not been fully explained in the PREM and other models. Is there a mathematical and logical relationship between the various layers of the earth's interior? Do they follow uniform mathematical rules? This paper believes there are uniform rules, which can be expressed by the \(2^n\) geometry model. The \(2^n\) geometry theory has derived the earth's inner core may have a boundary near 5574 km in March 2020\(^1\), and later the Thanh-SonPham. & Hrvoje Tkalčić team confirmed in 2023 there is a sphere with a radius of 650 km in the earth's inner core \(^1\), \(^2\) (5773 km from the ground). This paper deduces the radius is 647 km.

The \(2^n\) model is a new geometric physics theory, which can predict some currently undiscovered earth structures, or explain the current controversies and defects in seismic wave detection methods \(^10\). It can also explain the causes of major discontinuities, such as 'Gutenberg discontinuity' and 'Leiman discontinuity', from a new theoretical perspective, and explain the formation mechanism of various discontinuities with new physical and geometric ideas. In addition, the theory can also understand the earth's spherical structure and its internal logical from a new perspective, and predict and derive some new discontinuities to be confirmed.

2. Introduction to the Principle of \(2^n\) Geometry Method

A sphere, \(Q\) with a diameter of \(\varnothing\) is projected on a plane 'p' through the center of the sphere, which is a circle with a diameter of \(\varnothing\). We call this \(\varnothing_0\) starting ball' and the circle on the plane 'p' 'starting circle \(0\)'. Therefore, the diameter of sphere \(\varnothing_0\) divided by \(2^n\) is obtained as follows: \(\varnothing_n=\varnothing/2^n\), \(n=(0.1.2.3.4.5.6.7.8)\), and the diameter of \(\varnothing_0\) is \(\varnothing_0\). Thus, eight spheres are produced according to the diameter of \(\varnothing_0\) reduced to the power of \(2^n\): \(Q_1, Q_2, Q_3, Q_4, Q_5, Q_6, Q_7, Q_8\).

2.1 Dimensional expansion in the initial plane \(p\)

The \(\varnothing_0\)'s a contracted diameter on the same plane: a starting circle on the plane with a diameter of \(\varnothing_0\), \(\varnothing_0=2R\), \(R\) represents the radius, then
\[
\varnothing_0^2 = \varnothing / 2^n = 2R / 2^n \quad (1)\]
Each \( n \) corresponds to the diameter of a circle of \( \mathcal{O}_n \), which is constricted by \( 1/2 \) in turn each time. The circles produced by to contract \( 1-8 \) are called 'shrinking symmetrical circles'. We arrange the centers of these \( 1-8 \) shrinking circles' in a straight line on a plane in turn, and form a one-dimensional shrinking geometry of \( \mathcal{O}_n \) in the plane \( p \), as shown in Fig. 1a.

2.1.1 The distance \( l^{(\text{a or b})}_n \) from the tangent point \( l \) to the starting point \( \theta \) of a symmetrical circle:

\[
\begin{align*}
 l_n^{(\text{a})} &= 2\mathcal{O}(1-2^{n}) \\
 l_n^{(\text{b})} &= 2\mathcal{O}(1-2^{n}) + (\mathcal{O} \text{ or } 2\mathcal{O})(n\neq0)
\end{align*}
\]

2.1.2 The distance \( \mathcal{O}_n \) between the center \( O \) of the symmetric circle and the starting point \( \theta \):

\[
\begin{align*}
 \mathcal{O}_n^{(\text{a})} &= \frac{l_n^{(\text{a})}}{2} = \mathcal{O}(1-2^{n}) \\
 \mathcal{O}_n^{(\text{b})} &= \frac{l_n^{(\text{b})}}{2} = \mathcal{O}(1-2^{n}) + (\mathcal{O} \text{ or } 2\mathcal{O})(n\neq0)
\end{align*}
\]

2.1.3 Extended type (as shown in Fig. 1b): a starting circle with diameter \( \mathcal{O} \) on a plane, \( \mathcal{O}=2R, R \) represents the radius, then:

\[
\begin{align*}
 \mathcal{O}_n^{(\text{a})} &= 2^n \mathcal{O} = 2^{n+1}R \\
 \mathcal{O}_n^{(\text{b})} &= 2^n \mathcal{O} = 2^{n+1}R + \mathcal{O}(n\neq0,1)
\end{align*}
\]

2.1.4 The distance \( l_o \) of the tangent point \( l \) of a symmetric circle from the starting point \( \theta \):

\[
\begin{align*}
 l_o^{(\text{a})} &= 2^n \mathcal{O} \\
 l_o^{(\text{b})} &= 2^n \mathcal{O} + \mathcal{O}(n\neq0)
\end{align*}
\]

2.1.5 The distance \( \mathcal{O}_n \) of the center \( O \) of the symmetric circle from the starting point \( \theta \):

\[
\begin{align*}
 \mathcal{O}_n^{(\text{a})} &= l_o^{(\text{a})}/2 = 2^{n-1} \\
 \mathcal{O}_n^{(\text{b})} &= l_o^{(\text{b})}/2 = 2^{n-1} \mathcal{O} + \mathcal{O}/2(n\neq0)
\end{align*}
\]

Each \( n \) corresponds to a-type and b-type of \( (\mathcal{O}_n, l_o, \mathcal{O}_n) \), and in turn to double the extension. We call the circles generated by the 1~8 process 'extended symmetrical circles', referred to as symmetrical circles. Arrange the circles 0~8 in a plane \( p \) in turn, and their centers are on a straight line, to form the geometry of the one-dimensional extension of \( (\mathcal{O}_n, l_o, \mathcal{O}_n) \) in the plane (Fig. 1a and Fig. 1b). The model is divided into two types: 'a' and 'b', refer to \( a \) and \( b \) in Fig. 1 and Fig. 2.

3. The derivation method of the \( 2^n \) geometry principle of the earth and the construction and main discontinuity of the model

The whole earth is represented by \( E \), and the diameter of \( E \) is represented by \( \mathcal{O}, \mathcal{O}=2R_e \). First, in the plane through the equator, the earth \( (E) \) divide into two large circles 'BE' (hemisphere), \( \mathcal{O}_{BE}=\mathcal{O}/2=6371 \text{km} \). Then continue to divide \( BE \) into two parts, represented by \( WE \) (outer

![diagram](image1.png)

![diagram](image2.png)
hemisphere) and \( NE \) (inner hemisphere),
\[
\mathcal{O}_{WE} = \mathcal{O}_{NE} = R/2 = 3185.5 \text{km.}
\]
The layer from the discontinuity to 3185.5km call the outer sphere; the layer from 3185.5 to the center of the earth call the inner sphere (core).

The distance from the spherical symmetry circle within the outer sphere to the surface denote by the symbol \( 'l' \). Then, the surface interface variation formula can express as superscript \( (a \text{ or } b) \) representing different geometric space types, where \( n \) takes on values ranging from \( 0, 1, 2, 3, 4, 5, 6, 7, 8 \).

3.1 The formula for the spheres of hemispherical \( BE \) system with spheres of type \( 'a' \) and \( 'b' \):

\[
\begin{align*}
\mathcal{O}_{n}^a &= \mathcal{O}_{n}^b / 2^n = (R_\omega / 2^n)2^n \\
\mathcal{O}_{0}^a &= \mathcal{O}_{0}^b = R_\omega (2^8 + 1) \\
\mathcal{O}_{n}^b &= \mathcal{O}_{0}^b \times 2^{n-1} = R_\omega (2^8 + 1)2^{n-2}, (n \neq 0, 1) \\
P_n^a &= (R_\omega / 2^n)^2 \\
P_n^b &= R_\omega (2^8 + 1)2^n + R_\omega (2^8 + 1), (n \neq 0) \\
O_n^a &= P_{n-1}^a + (P_{n-1}^b / 2) \\
O_n^b &= \mathcal{O}_{0}^\omega / 2 \\
O_n^b &= (P_n + P_{n-1}) / 2, (n \neq 0)
\end{align*}
\]

4. Result

4.1 Whole sphere discontinuity

\[ E = BE + \text{mirrored } BE = (\text{hemispherical plane boundary} + \text{mirrored hemispherical plane boundary}), \]
\[
\begin{align*}
2^n \text{ geometry space of global plane}, \\
\mathcal{O}_{n}^a &= \mathcal{O}_{n}^b / 2^n = (2R_\omega / 2^n)2^n = 2R_\omega / 2^{8-n} \\
\mathcal{O}_{0}^a &= \mathcal{O}_{0}^b = 2R_\omega (2^8 + 1) \\
\mathcal{O}_{n}^b &= \mathcal{O}_{0}^b \times 2^{n-2} = 2R_\omega (2^8 + 2)2^{n-2}, (n \neq 0, 1, 2) \\
P_n^a &= 2R_\omega / 2^{8-n} \\
P_n^b &= 2R_\omega (2^8 + 2)(2^{n-1} + 1), (n \neq 0) \\
O_n^a &= \mathcal{O}_{0}^\omega / 2^{8-n} \\
O_n^b &= P_{n-1}^a + (P_{n-1}^b / 2) = 3R_\omega / 2^{9-n}, (n \neq 0) \\
O_d^b &= 2R_\omega (2^8 + 2) \\
O_d^b &= 2R_\omega (2^8 + 2)2^n + 2R_\omega (2^8 + 2) = 3R_\omega (2^8 + 2) \\
O_n^b &= (P_n + P_{n-1}) / 2, (n \neq 0)
\end{align*}
\]

Table 1. Lists the position and attribute (complex number state) of the global \( E \) type \( (a, b) \) starting circle and symmetrical circle. The position of the starting circle center of \( E \) determines the position of each symmetrical circle, as well as the edge \( (l_{\text{outer}}, l_{\text{interior}}) \) and center \( (O) \) of each symmetrical circle. Among them, \( 'a' \) represents the positive real number domain attribute, the outer edge of which has discontinuity and is easy to detect by seismic waves. There are obvious discontinuous or transition zones on both sides of the one-dimensional straight line projection, and the density on both sides of the center is small and uniform. While \( '-a' \) represents the negative real number domain attribute, the density of the center of the number domain is greater than that on both sides, and is more easily detected by seismic waves. The above is the change rule of \( 2^n \) geometric principal sequence. When \( n \) is \( 0 \sim 8 \), the entire earth divide into 9 large regions of starting circle + symmetrical circle.
4.2 Hemisphere BE

The hemisphere is the differentiation of the next layer inside the global layer. Taking the earth's center as the boundary, the hemisphere divides the earth into two symmetrical symmetrical circles (as shown in Figure 1 and Figure 2) in the equatorial plane. The calculation formulas for the diameter, position and main boundary of the symmetrical circles are as follows: (7)\textsuperscript{a}, (7)\textsuperscript{b}, (8)\textsuperscript{a}, (8)\textsuperscript{b}, (9)\textsuperscript{a}, (9)\textsuperscript{b}

<table>
<thead>
<tr>
<th>n</th>
<th>( P_n )</th>
<th>( \Omega_n^a )</th>
<th>( P_{n\text{outer}} \sim O_n^a \sim P_{n\text{interior}} )</th>
<th>( P_n )</th>
<th>( \Omega_n^b )</th>
<th>( P_{n\text{outer}} \sim O_n^b \sim P_{n\text{interior}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>12742</td>
<td>12742</td>
<td>0 ( \sim 6371 \sim 12742(0) )</td>
<td>12692.6</td>
<td>6321.6</td>
<td>6371 ( \sim 3210-12693(49.38) )</td>
</tr>
</tbody>
</table>

Table 2. Hemispheric layer, diameter, position, main boundary and attributes (in the complex state) of type a and b. The main boundary determines the position of the starting circle relative to each symmetrical circle, and determines the outer edge, center and inner edge generated by the symmetrical circle when rotating around the system center.

4.3 Secondary differentiation of hemispheric BE

The part close to the outer circumference call the outer differentiation circle \( WE \), and the part close to the center of the sphere call the inner differentiation circle \( NE \). The expansion and contraction of \( 2^n \) geometric space are reciprocal processes, and the change of the internal second-order boundary of the outer sphere is the reverse process of the change of the main order.
Here, only the second-order boundary of the inner differentiation circle is introduced. The b-type change of the internal second-order circle is slightly different from the main order. The boundary of the internal second-order circle is represented by $\gamma \mathcal{O}$, where the subscript ‘$\gamma$’ represents the second-order boundary, and the distance from the surface to the boundary is represented by $\gamma I_{0}$.

\[ \gamma \mathcal{O}^a_n = (R / 2^b) 2^n - (n\#0), \gamma \mathcal{O}^b_n = R / 2^b \]

(12) a

\[ \gamma \mathcal{O}^b_n = (R / 2^b) 2^n - (n\#0) \]

(12) b

\[ \gamma \mathcal{O}^b_n = (R / 2^b) 2^n - (n\#0) \]

(13) a

\[ \gamma \mathcal{O}^b_n = (R / 2^b) 2^n - (n\#0) \]

(13) b

\[ \gamma \mathcal{O}^b_n = (\gamma \mathcal{O}^b_n + \gamma \mathcal{O}^b_{n+1}) / 2, (1)^n \]

(14) a

\[ \gamma \mathcal{O}^b_n = (\gamma \mathcal{O}^b_n + \gamma \mathcal{O}^b_{n+1}) / 2, (1)^n, (n\#0) \]

(14) b

\[ n = (1, 2, 3, 4, 5, 6, 7, 8, 0) \]

The attribute ‘$a$’ is a demarcation attribute, indicating there is a discontinuity at this location, and ‘$-a$’ indicates there are transition layers on both sides of this location. Table 3. The primary boundaries of type a and type b of hemispheric NE (in the real state). The central boundaries of type a and type b of hemispheric NE (in the real state) calculate by $\gamma \mathcal{O}^a_n$ formula (14) a and yobn formula (14) b respectively. The calculation formula of ylbn is (13) a, (-1) b; the calculation formula of ylb0n is (12) b, where $\gamma \mathcal{O}^a_n$ is (12) b and y0bn is (12) b.

<table>
<thead>
<tr>
<th>$\gamma \mathcal{O}^a_n$ boundary</th>
<th>$\gamma \mathcal{O}^b_n$ boundary</th>
<th>$\gamma \mathcal{O}^b_{n+1}$ boundary</th>
<th>$\gamma \mathcal{O}^b_{n+2}$ boundary</th>
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</tr>
<tr>
<td>$\gamma \mathcal{O}^a_n$ Main boundary</td>
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<td>$\gamma \mathcal{O}^b_{n+1}$ Main boundary</td>
<td>$\gamma \mathcal{O}^b_{n+2}$ Main boundary</td>
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<td>$\gamma \mathcal{O}^b_{n+1}$ outdoor ~ $\gamma \mathcal{O}^b_{n+1}$ interior</td>
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</tr>
<tr>
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</tr>
</tbody>
</table>

5. Differentiation and second-order adjustment values of $\gamma \mathcal{O}^a_n$, $\gamma \mathcal{O}^b_n$, $\gamma \mathcal{O}^a_n$, $\gamma \mathcal{O}^b_n$ boundary of each layer of a and b spheres.

5.1 Overlapping dominance principle: various data sets of starting circles (as shown in Fig. 3 and Fig. 4) are arrange according to position. The more data overlap, the easier the boundary formed here is to detect.

Fig. 2 Shows the set (discontinuous group) of $\gamma l_{00}$, $\gamma l_{00}$, $\gamma O_{00}$ and $\gamma O_{00}$ in the spatial position of the starting circles of each spherical layer. The red numbers represent the distances between the starting circles of E, BE, WE and NE, $\gamma O_{00}$ and $\gamma O_{00}$, and the surface from far to near. The yellow numbers represent the corresponding position data at the center of each spherical layer $\gamma O_{00}$, indicating that there is an easy-to-detect boundary or obvious transition zone at this position. The more overlapping numbers, the stronger the detectability; the gray part indicates the data has no corresponding position, and only has a weak boundary that is not easy to detect (need further study). The magenta markings indicate the adjacent positions on both sides that may resonate near the middle position between adjacent data, thus enhancing the detectability of the boundary at the middle position. These elements form a resonant set, in which the data about large form a resonant boundary and transition zone that is easier to detect than the data of other elements.
5.2 $O^a_m, O^b_m, I^a_m, I^b_m, O^a_n, O^b_n$ Secondary adjustment

According to the $2^n$ geometry calculation, there is a certain degree of position offset in the center position of the symmetrical circle, and the offset result is the second-order adjustment value.

$\gamma O^a_m \pm O^a_n/2^3$  \hspace{1cm} (15)$^a$

$\gamma O^b_m \pm O^b_n/2^3$  \hspace{1cm} (15)$^b$

$\gamma O^a_m \pm O^b_n/2^3 \times 3$  \hspace{1cm} (16)$^a$

$\gamma O^b_m \pm O^a_n/2^3 \times 3$  \hspace{1cm} (16)$^b$

$\gamma O^a_m \pm O^a_n/2^2$  \hspace{1cm} (17)$^a$

$\gamma O^b_m \pm O^b_n/2^2$  \hspace{1cm} (17)$^b$

Fig. 3 The $2^n$ geometry derived $E+WE+BE+NE$ composite data with the center position of symmetrical circle and the second-level adjustment value. The same color classify into one category, and the color also represents the degree of overlap of the center and edge position data of various levels of symmetrical circle. $NE^m_1, P^o_1$ and $NE^o, P^m_1$ regions are the re-division regions of the earth's core (5723.6~5726.4km), $NE^m_2, P^o_2$ and $NE^o, P^m_2$ regions are the inner and outer core transition region II, to explore the Lyman surface. $NE^m_1, P^o_1$ and $NE^o, P^m_1$ regions are the inner and outer core transition region I, about 3180 or 4778km (controversial). $WE^m_2, WE^o_3, BE^m_2, BE^o_3$ and $E^m_P, E^o_P$ together constitute the nuclear-mantle demarcation region in the traditional sense [13]. Because the tectonic causes of these locations are complex, and they are acted upon by a variety of factors, all of which lead to complex tectonics here.
6. Discussion

The physical properties of the data in Figure 2 and Figure 3 complement each other. The
locations with more overlaps are easy to detect, and the locations without data overlaps are difficult to detect without discontinuities or discontinuities. The data in Figure 2 derive from Figure 3. Some of the data in each data group in Figure 3 are close to the proven discontinuities, and some are not. The data in these unproved data groups need further research to confirm whether they have physical boundaries. Whether the data in the data group has the nature of recessive or dominant (easy to detect or difficult to detect or does not exist) needs further research to confirm.

7. Summary

The 2nd geometry principle is used to model and derive the earth's internal structure model, and the model established by seismic data is compared, and the main discontinuity locations are similar.

According to the statistical results in Fig. 2 and Fig. 3, the most overlapping area is the boundary area between the inner and outer nuclei (near the Lehmann discontinuity), 11 times; The next is the core mantle boundary near the Gutenberg discontinuity, 7 times. The upper and lower core mantle boundary areas also overlap 7 times, and then the core boundary overlaps 5 times.

According to the 2nd geometry principle, these multiple overlapping locations form the area of the main interface. Because of the multiple data overlap, it is easier to detect the Goldberg discontinuity and the Lehmann discontinuity, which overlap leads to the material composition of the location significantly different from the surrounding area.

Due to the multiple overlap of these positions, the material composition of the position is significantly different from that of the periphery, and it is easy to detect.

In no data overlap, we observed three adjacent and near arithmetic sequences, such as 796–818–839.6km. This leads to data resonance, and there is a co seismic overlap with the symmetrical circle data near the symmetrical circle position, thus forming a relatively secondary weak discontinuity.

The closer to the Surface, the easier it is to detect this type of discontinuity (such as Moho discontinuity, upper and lower mantle demarcation).

Compared with traditional earth structure models (such as PREM model), 2nd geometry theory derives some structural layers that have not detect by traditional methods, which need to test experimentally to confirm their existence.

Reference

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