# Adaptive RBF Control Based on Predefined-Time Stability For Flexible Manipulator

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**Abstract.** In view of the background that customs use flexible manipulator to perform remote dangerous goods detection operations, this paper proposes an adaptive RBF control method based on the definition of nonlinear robot system and predetermined time stability to meet the requirements of customs robot remote operation tasks. The design of the adaptive RBF controller can compensate for the uncertainties in the robot dynamics, and the adjustable parameters make the design of the control system more flexible, ensuring accurate tracking control in the presence of system model uncertainty and disturbances. The Lyapunov method is used in this research to demonstrate the stability of the control system, and robot tracking control simulation and experimentation are used to confirm the efficacy of the aforementioned approach.

**Keywords:** Adaptive RBF; Predefined-Time Stability; Remote Operation; Customs; Robots.

## 1. Introduction

With the acceleration of globalization, the number of cross-border goods is increasing. Traditional manual inspection methods often cannot meet the dual needs of safety and efficiency. Therefore, introducing robotics technology into the field of dangerous goods inspection, especially through remote operation, has become an innovative and effective solution. Remote operation can significantly improve safety. When conducting dangerous goods inspection, potential risks and uncertainties are often worrying. By using robots for remote operation, customs officers can monitor and control robots in a safe environment and avoid direct contact with possible dangerous substances. Therefore, the motion control of robots needs to be fast and accurate to ensure efficient completion of operational tasks.

Sliding mode control has strong robustness and anti-interference ability [1-9] and has been widely used in trajectory tracking of nonlinear systems, but there will be obvious jitter phenomenon. Since the convergence time of these sliding mode control methods cannot be determined in advance by calculating the control parameters, they are usually called finite time control methods. To this end, several predetermined time stable control systems are studied. Zhang et al. proposed a new robust scheduled time tracking control method for the global tracking control problem of robots with uncertainty and external disturbances [10]. Aldo et al. designed a scheduled time stabilization dynamic controller based on the inherent passivity of the robot dynamic structure [11]. Jia et al. proposed an adaptive scheduled time SMC for nonlinear systems, which can accelerate the convergence speed of the system [12]. The introduction of these methods has promoted the development of tracking control with time constraints. Traditional robust control cannot select the stable time as an accurate control parameter. Therefore, it is meaningful to study a simple and robust scheduled time tracking control method.

This paper combines the scheduled time stability theorem with the adaptive RBF sliding mode control algorithm, and designs a new sliding surface and controller on this basis, which can ensure that the customs robot system can complete the operation task quickly and accurately within the set scheduled time. The stability of the robot system under the action of the scheduled time and adaptive RBF controller is proved by Lyapunov function.

## 2. Preparation

### 2.1 Predetermined time Stability

For predetermined time constants $T_a$ , The origin of a system $\dot{x} = f(x)$ is referred to as fixed-time stable if both the stable time function and it are fixed-time stable $T : R_n \to R$ The following conditions are met:

$$T(x_0) \le T_a \ ,\forall x_0 \in \mathbb{R}^n. \tag{1}$$

If the above conditions are met, $T_a$ is called the predetermined time.

For a system, if there satisfies the following conditions:

$$\dot{V} \ \le \ -\frac{\pi\left(\delta V^{1-\frac{n}{2}} + \varepsilon V^{1+\frac{n}{2}}\right)}{nT_c\sqrt{\delta\varepsilon}} \tag{2}$$

After that, the system is stable for the specified amount of time. $T_c > 0$ is the predetermined time and $0 < n < 1, \delta, \varepsilon > 0.$

### 2.2 Robot dynamics model

The robot system is described by the Euler-Lagrangian form as follows:

$$H(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \tau + d \tag{3}$$

Among them, $q,\dot{q},\ddot{q} \in \mathbb{R}^{n\times 1}$ indicates the generalized joint coordinates' angular position, angular velocity, and angular acceleration.; $H(q) \in \mathbb{R}^{n\times n}$ symbolizes the matrix of positive definite inertia; $C(q,\dot{q}) \in \mathbb{R}^{n\times n}$ is Coriolis and centrifugal torque impacts of matrix; $G(q) \in \mathbb{R}^{n\times 1}$ symbolizes the gravity effects vector; $\tau \in \mathbb{R}^{n\times 1}$ is the input torque vector; $d \in \mathbb{R}^{n\times 1}$ represents the internal uncertainty and the bounded external disturbance vector.

Taking into account the uncertainty components of the model itself, the following equation is defined:

$$\begin{gathered} H(q) = H_i(q) + \hat{H}(q) \\ C(q,\dot{q}) = C_i(q,\dot{q}) + \hat{C}(q,\dot{q}) \\ G(q) = G_i(q) + \hat{G}(q) \end{gathered} \tag{4}$$

The dynamic equations are expressed as follows:

$$H_i(q)\ddot{q} + C_i(q,\dot{q})\dot{q} + G_i(q) = \tau + d_i \tag{5}$$

The following Jacobian matrix $J_A$ is used to transform from joint space to workspace, that is, $\dot{x} = J_A \cdot \dot{q}, \ddot{x} = \dot{J}_A J_A^{-1}\dot{x} + J_A\ddot{q}$ .is further written as:

$$\hat{\hat{H}}_i(q)\ddot{x} + \hat{\hat{C}}_i(q,\dot{q})\dot{x} + \hat{\hat{G}}(q) = F_x + F_d \tag{6}$$

In:

$$\begin{aligned}&\hat{\hat{H}}_i(q)=J_A^{-T}H_i(q)J_A^{-1}\\&\hat{\hat{C}}_i(q,\dot{q})=J_A^{-T}\left(C_i(q,\dot{q})-H_i(q)J_A^{-1}\dot{J}_A\right)J_A^{-1}\\&\hat{\hat{G}}_i(q)=J_A^{-T}G_i(q),F_x=J_A^{-T}\tau,F_d=J_A^{-T}d_i\end{aligned}\tag{7}$$

## 3. Sliding surface and controller design

In order to achieve the predetermined time stability by sliding mode, this paper designs the following sliding surface:

$$s=\dot{\rho}+\frac{\rho\pi\left(\delta\|\rho\|^{-n}+\varepsilon\|\rho\|^{n}\right)}{nT_c\sqrt{\delta\varepsilon}}+\alpha\rho\tag{8}$$

Wherein, $\rho=x-x_d$ is the position tracking error, $x_d$ is the expected value of $x$ , $n\in(0,1)$ and $\delta\in(0,1)$ , $\varepsilon\in(0,1)$ , $\alpha>0$. $T_c$ are the predetermined times.

Theorem 1: If a sliding surface (8) is used by the error system , it converges to zero within a predetermined time $T_c$.

Proof: When the sliding surface has a position error $S(t)=\{\rho|s(\rho)=0\}$, therefore:

$$\dot{\rho}=-\left(\frac{\rho\pi\left(\delta\|\rho\|^{-n}+\varepsilon\|\rho\|^{n}\right)}{nT_c\sqrt{\delta\varepsilon}}+\alpha\rho\right)\tag{9}$$

If $V_\rho=\frac{1}{2}\rho^T\rho=\frac{1}{2}\|\rho\|^2$,So

$$\begin{aligned}\dot{V}_\rho&=\rho^T\dot{\rho}\\&=-\frac{\rho\pi\left(\delta\|\rho\|^{2-n}+\varepsilon\|\rho\|^{2+n}\right)}{nT_c\sqrt{\delta\varepsilon}}-\alpha\|\rho\|^2\\&=-\frac{\rho\pi\left(\delta\|\rho\|^{2-n}+\varepsilon\|\rho\|^{2+n}\right)}{nT_c\sqrt{\delta\varepsilon}}-\alpha\|\rho\|^2\\&\le-\frac{\rho\pi\left(\delta\|\rho\|^{2-n}+\varepsilon\|\rho\|^{2+n}\right)}{nT_c\sqrt{\delta\varepsilon}}\\&=-\frac{\rho\pi\left(\delta V_\rho^{1-\frac{n}{2}}+\varepsilon V_\rho^{1+\frac{n}{2}}\right)}{nT_c\sqrt{\delta\varepsilon}}\end{aligned}\tag{10}$$

A new variable $P$ is introduced in the controller and expressed as follows:

$$P=x_d-\frac{\rho\pi\left(\delta\|\rho\|^{-n}+\varepsilon\|\rho\|^{n}\right)}{nT_c\sqrt{\delta\varepsilon}}-\alpha\rho\tag{11}$$

In the sliding plane, the equation (5) is expressed as follows:

$$\hat{\hat{H}}_i(q)\dot{s}+\hat{\hat{C}}_i(q,\dot{q})s+\hat{\hat{H}}_i(q)\dot{P}+\hat{\hat{C}}_i(q,\dot{q})P+\hat{\hat{G}}_i(q)=F_x+F_d\tag{12}$$

$$F_x = \hat{\hat{H}}_i(q)\dot{P} + \hat{\hat{C}}_i(q,\dot{q})P + \hat{\hat{G}}_i(q) - \frac{s\pi\left(\delta\|s\|^{-n} + \varepsilon\|s\|^{n}\right)}{nT_c\sqrt{\delta\varepsilon}} - w^T\gamma(x) \tag{13}$$

Among them, $w$ is the weight matrix of RBFNN, $\gamma(x)$ is its foundational role. The input $x$ is a multidimensional vector: $[x \quad \dot{x} \quad \ddot{x} \quad x_d \quad \dot{x}_d \quad \ddot{x}_d]$.

If the estimated value of RBFNN is expressed as $F_d = \hat{w}^T\gamma(x)$, then the estimated error value can be expressed as $\tilde{w} = \hat{w} - w$. In this way, the adaptive expression of the weight can be obtained as $\dot{\hat{w}} = \dot{\tilde{w}} = \beta\gamma(x)s^T$. Among them, $\beta$ is a diagonal matrix that is positive definite, the following Lyapunov function is established:

$$\bar{V} = \frac{1}{2}s^T\hat{\hat{H}}_i(q)s + \frac{1}{2}tr\left(\tilde{w}^T\beta^{-1}\tilde{w}\right) \tag{14}$$

Among $\frac{1}{2}s^T\hat{\hat{H}}_i(q)s \le \mu\|s\|^2$, $0 < \mu < 1$.

For the above equation(14), we can get:

$$\dot{\bar{V}} = \frac{1}{2}s^T\dot{\hat{\hat{H}}}_i(q)s + s^T\hat{\hat{H}}_i(q)\dot{s} + tr\left(\tilde{w}^T\beta^{-1}\dot{\tilde{w}}\right) \tag{15}$$

Combined with the properties of the robot system, $\dot{\hat{\hat{H}}}_i(q) - 2\hat{\hat{C}}_i(q,\dot{q})$ represents the antisymmetric matrix, we can get:

$$\begin{aligned}
\dot{\bar{V}} &= \frac{1}{2}s^T\dot{\hat{\hat{H}}}_i(q)s + s^T\hat{\hat{H}}_i(q)\dot{s} + tr\left(\tilde{w}^T\beta^{-1}\dot{\tilde{w}}\right) \\
&= s^T\left(-\frac{s\pi\left(\delta\|s\|^{-n} + \varepsilon\|s\|^{n}\right)}{nT_c\sqrt{\delta\varepsilon}}\right) - s^T\tilde{w}\gamma(x) + tr\left(\tilde{w}^T\beta^{-1}\dot{\tilde{w}}\right) \\
&\le -\frac{\pi\left(\delta\|s\|^{2-n} + \varepsilon\|s\|^{2+n}\right)}{nT_c\sqrt{\delta\varepsilon}} \\
&\le -\frac{\pi\left(\delta\bar{V}^{1-\frac{n}{2}} + \varepsilon\bar{V}^{1+\frac{n}{2}}\right)}{nT_c\sqrt{\delta\varepsilon}}
\end{aligned} \tag{16}$$

According to equation (16) and $0 < \delta, \varepsilon < 1, n \in (0,1)$, we can conclude that Theorem 1 is satisfied and the predetermined time stability of the position error can be promised within the given predetermined time .

## 4. Numerical simulation

The tracking performance is compared to the current robust control approaches using the following two-DOF robot model. The dynamic model used by the robot in the simulation is as follows:

$$H_i(q)=\begin{bmatrix} M_1+M_2+2M_3\cos q_2 & M_2+M_3\cos q_2 \\ M_2+M_3\cos q_2 & M_2 \end{bmatrix}$$
$$C_i(q)=\begin{bmatrix} -M_3\dot{q}_2\sin q_2 & -M_3(\dot{q}_1+\dot{q}_2)\sin q_2 \\ M_3\dot{q}_1\sin q_2 & 0.0 \end{bmatrix} \quad (17)$$
$$G_i(q)=\begin{bmatrix} M_4 g\cos q_1+M_5 g\cos(q_1+q_2) \\ M_5 g\cos(q_1+q_2) \end{bmatrix}$$

The following are the robot's settings. The $M_i$ value of $M_i=B_i+\kappa L_i$. is We take $M_i=[M_1,M_2,M_3,M_4,M_5]^T$ , $B_i=[B_1,B_2,B_3,B_4,B_5]^T$ , $L_i=[l_1^2,l_2^2,l_1l_2,l_1,l_2]^T$ , $l_1=l_2=1$ as the length of the corresponding joint, where $B_i$ is the execution parameter variable of the robot manipulator. We consider $\kappa=0.5, B_i=[1.66,0.42,0.63,3.75,1.25]^T, g=9.8N/s^2$ . $T_c=0.6$ , $\delta=0.3$ , $\alpha=0.9$ , $n=0.5$ . The expected trajectory is set to $[1-0.2*\cos(\pi t) \quad 1+0.2*\sin(\pi t)]$, Joints 1 and 2 have their starting positions set to 0.83 and 1.08.

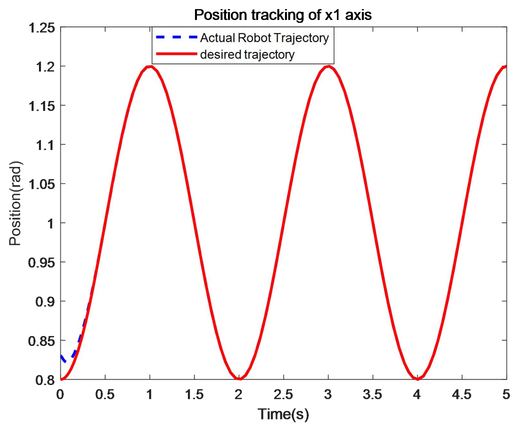


Fig.1 The x1 axis trajectory tracking

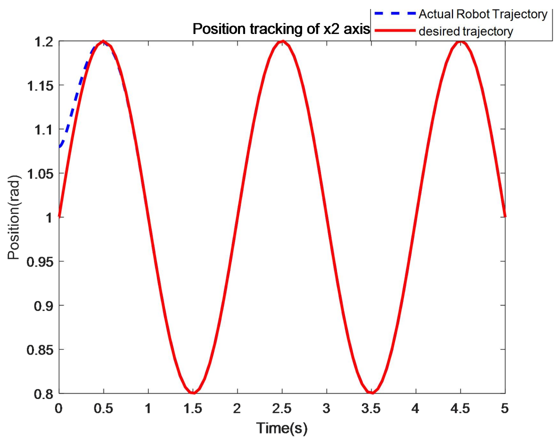


Fig.2 The x2 axis trajectory tracking

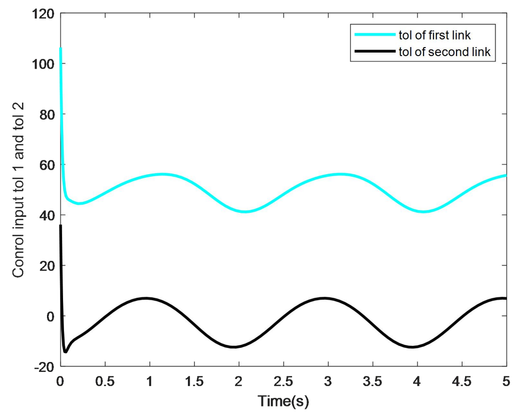


Fig.3 Input torques of the joint in the paper

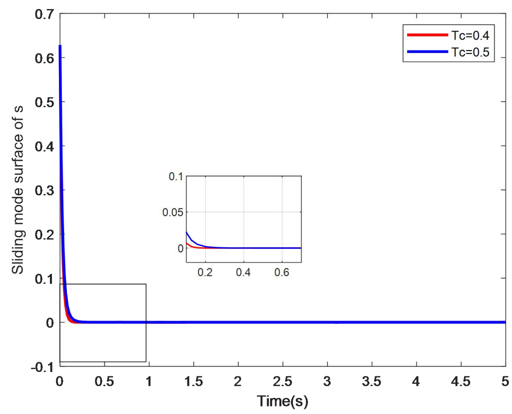


Fig.4 Sliding mode surface under different predefined-time

From Figure 1 and Figure 2, we can see that stable and accurate tracking can be achieved within the scheduled time of 0.6s. Figure 3 is the control input result of the scheduled time sliding mode control, and Figure 4 is a comparison of the duration required to arrive at the sliding surface under different scheduled times. In summary, it can be concluded that the new scheduled time sliding mode control scheme proposed has faster convergence speed, shorter convergence time, and better robustness.

## 5. Conclusion

For the customs robot remote control system, this paper designs a scheduled time adaptive RBF sliding mode controller based on the definition of nonlinear multi-joint manipulator system and scheduled time stability to resist the interference of model uncertainty and external disturbances and achieve precise tracking control. Through numerical simulation and real robot experiments, it can be concluded that the control method designed in this paper has a faster convergence speed and shorter convergence time, and this simultaneously confirms the viability and efficiency of the control strategy put forward in this research.

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